Complex Decision-Aided Maximum-Likelihood Phase Noise and Frequency Offset Compensation for Coherent Optical Receivers

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Abstract *We present a novel complex decision-aided maximum-likelihood receiver for joint phase noise and frequency offset compensation, with automatic on-line filter weight adaptation using a least-sum-of-squared-error criterion. Frequency offset is acquired quickly and compensated perfectly for a complete frequency offset range of* ± 1 *times the symbol rate.*

Introduction

A key digital signal processing function in coherent systems is to recover the phase and frequency of the carrier in the digital domain rather than using optical phase-locked loops. This enables the use of a free-running local oscillator (LO) laser. The frequency offset, Δf, between the transmitter and LO lasers over their lifetime can be as large as ± 5 GHz[1]. However, the popular block Mth power carrier estimation (CE) requires the frequency offset to be kept below 10 MHz[2]. We previously proposed a decision-aided, maximum-likelihood (DA ML) CE[3,4]. Unfortunately, DA ML CE has a limited frequency offset tolerance, incurring a 1.8 dB signal-to-noise ratio (SNR) per bit penalty for an increase of Δf to 100 MHz at 1.27 MHz laser linewidth[5]. In this paper, we present a new complex DA ML CE for joint phase noise and frequency offset compensation by extending upon the DA ML algorithm to incorporate frequency offset estimation (FOE) capability.

Compared to the pioneering differential FOE technique[6], complex DA ML CE avoids the nonlinear $\arctan(\cdot)$ operation and the Mth power to remove data modulation. Consequently, our receiver is not confined to M-ary phase shift keying (MPSK) formats. Furthermore, a complete frequency offset compensation range of $\pm R$ which does not shrink with higher-order modulation formats is achieved. Here, R is the symbol rate. Our complex DA ML CE attains perfect frequency offset compensation without any knowledge of the spectral height of the additive white Gaussian noise (AWGN), laser linewidth or frequency offset. Additionally, no phase unwrapping is required and our complex DA ML CE is computationally linear.

Operating principle

Filtering the received waveform through a matched filter and sampling at the right time instants yields[5] $r(k) = m(k)\exp\left[j\left(2\pi\Delta f T k + \theta(k)\right)\right] + n(k)$. Here, $m(k)$ is the data symbol at time kT (T is symbol duration) and $\{n(k)\}$ is the complex AWGN with zero mean and variance σ_n^2. SNR per bit is $\gamma_b = \mathrm{E}[|m(k)|^2]/(\sigma_n^2 \log_2 M)$, where $|\cdot|$ and $\mathrm{E}[\cdot]$ denote modulus operation and statistical expectation, respectively. Laser phase noise is modeled as a random walk process $\theta(k) = \theta(k-1) + \eta(k)$. The $\{\eta(k)\}$ is a set of independent and identically distributed Gaussian random variables, each with mean zero and variance $\sigma_p^2 = 2\pi\Delta\nu T$. Here, $\Delta\nu$ is the combined 3-dB laser linewidth of transmitter and LO laser.

In DA ML CE, a reference phasor (RP) of the carrier was formed using the immediate past L received signals as

$$V(k+1) = C(k)\sum_{l=1}^{L} r(k-l+1)\hat{m}^*(k-l+1) \quad (1)$$

where $C^{-1}(k) = \sum_{l=1}^{L}|\hat{m}(k-l+1)|^2$ and $\hat{m}(k)$ is the receiver's decision on the kth symbol. Superscript * denotes conjugate. The phase noise is assumed to be time invariant at least over an interval longer than LT. In the presence of a frequency offset, consecutive modulation-wiped-off filter input terms $\{y(k-l+1) = r(k-l+1)\hat{m}^*(k-l+1)\}_{l=1}^{L}$ will be offset by an additional phase rotation of $2\pi\Delta f T$. To account for this constant carrier rotation in our RP computation, we suggest multiplying each $y(k-l+1)$ term by $exp(j2\pi l\Delta f T)$. The filter input terms will then be phase-aligned during the subsequent vector summation process and thus yield a reliable RP for the next time point. We hence propose here a new RP $V'(k)$ formed as

$$V'(k+1) = C(k)\boldsymbol{w}^T(k)\boldsymbol{y}(k) \quad (2)$$

where $\boldsymbol{y}(k) = [y(k),\cdots,y(k-L+1)]^T$, $\boldsymbol{w}(k) = [w_1(k),\cdots,w_L(k)]^T$, and superscript T denotes transpose. The weight $w_l(k)$ is complex and will be an estimate of $exp(j2\pi l\Delta f T)$, functioning to rotate the $y(k-l+1)$ term. Filter (2) operates without the knowledge of SNR, laser phase noise, or frequency offset statistics. Since Δf is unknown, we propose to choose the weight

Tab. 1: Recursive weight vector update algorithm

Initialize recursive algorithm at time $k = 0$

1. $\boldsymbol{w}(0) = [1_1, 0_2, \cdots, 0_L]^T$
2. $\boldsymbol{R}^{-1}(0) = \delta^{-1}\boldsymbol{I}$; here δ^{-1} is a small positive constant and $\boldsymbol{I}$ is an identity matrix

For each time instant $k \geq 1$

1. Compute intermediate vector, $\boldsymbol{u}(k)$
   $\boldsymbol{u}(k) = C(k-1)\boldsymbol{R}^{-1}(k-1)\boldsymbol{y}^*(k-1)$
2. Compute gain vector, $\boldsymbol{g}(k)$
   $$\boldsymbol{g}(k) = \frac{\boldsymbol{u}(k)}{1 + C(k-1)\boldsymbol{y}^T(k-1)\boldsymbol{u}(k)}$$
3. Compute *a priori* estimation error, $\xi(k)$
   $$\xi(k) = \frac{r(k)}{\widehat{m}(k)} - C(k-1)\boldsymbol{w}^T(k-1)\boldsymbol{y}(k-1)$$
4. Update weight vector, $\boldsymbol{w}(k)$
   $\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \boldsymbol{g}(k)\xi(k)$
5. Recursively invert correlation matrix
   $\boldsymbol{R}^{-1}(k) = \boldsymbol{R}^{-1}(k-1) - \boldsymbol{g}(k)\boldsymbol{u}^H(k)$

vector $\boldsymbol{w}(k)$ adaptively at each time k based on the observations $\{r(l), 0 \leq l \leq k\}$ to minimize the cost function $J(k)$, where

$$J(k) = \sum_{l=1}^{k} \left| \frac{r(l)}{\widehat{m}(l)} - C(l-1)\boldsymbol{w}^T(k)\boldsymbol{y}(l-1) \right|^2 . \quad (3)$$

The cost function (3) forces $V'(k)$ to track the modulation-wiped-off received signal sample given by $r(k)/\widehat{m}(k)$. Solving $\partial J(k)/\partial \boldsymbol{w}^*(k) = 0$ yields the normal equation $\boldsymbol{a}(k) = \boldsymbol{R}(k)\boldsymbol{w}(k)$, where $\boldsymbol{a}(k)$ is an L-by-1 time-average cross-correlation vector. To get $\boldsymbol{w}(k)$, we have to invert the L-by-L time-average correlation matrix $\boldsymbol{R}(k)$ at each time k. We can obtain both $\boldsymbol{R}^{-1}(k)$ and $\boldsymbol{w}(k)$ recursively using the *matrix inversion lemma*[7] as summarized in Tab. 1, where superscript H denotes conjugate transpose. The inversion of $\boldsymbol{R}(k)$ is now replaced at each step by a simple scalar division. Hence, the need to store the entire past observed data is eliminated and computational complexity is greatly simplified. The RP $V'(0)$ is initialized to 1 and subsequently obtained from (2). Finally, the received signal $r(k)$ is de-rotated by $V'^*(k)$ before being fed into the optimum symbol-by-symbol detector

$$\widehat{m}(k) = \arg \max_{0 \leq i \leq M-1} \mathrm{Re}[r(k)V'^*(k)m_i^*] . \quad (4)$$

Results and discussion

A filter block length of L = 5 and a single-polarization signal with a baud rate of 20 Gbaud is maintained in all ensuing Monte Carlo simulations. An initial preamble of 50 known symbols is used to aid in reference acquisition and the filter operates in decision-directed mode subsequently. Differential encoding is used to prevent error propagation due to decision errors. Fig. 1(a) plots the adaptation of the angle of the complex weights $\{w_l(k)\}_{l=1}^{L}$, averaged over 500 runs at γ_b = 10 dB. A quadrature PSK (QPSK) signal at $\Delta\nu$ = 2 MHz and a frequency offset of 2 GHz is used. The angle of the weight $w_l(k)$ correctly estimates and converges quickly to the actual value of $2\pi l \Delta f T$. Therefore, the complex weight $w_l(k)$ indeed rotates $y(k-l+1)$ by $2\pi l \Delta f T$, ensuring a set of phase-aligned filter inputs during the subsequent vector summation process. It should be emphasized that the optimum complex weights $\{w_l(k)\}_{l=1}^{L}$ can respond to changing channel conditions, as it depends on the observations $\{r(k)\}$.

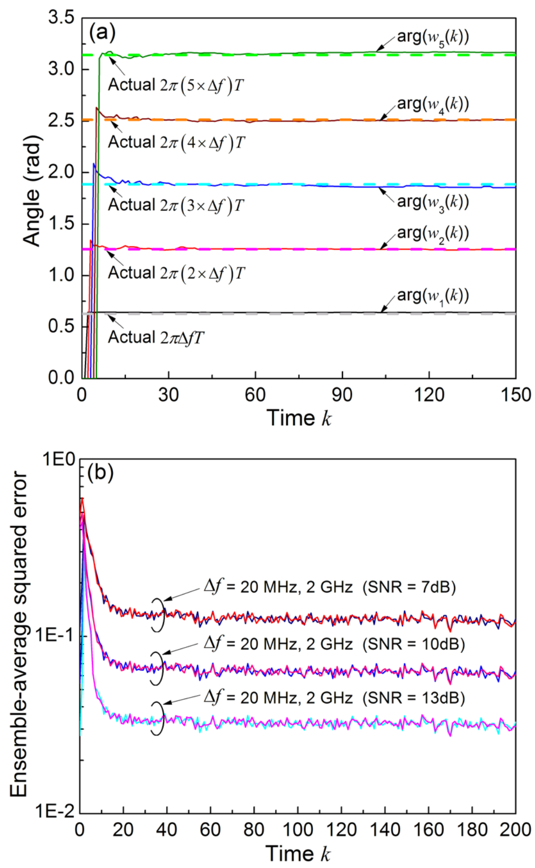

Fig. 1: (a) Adaptation process of $\arg(w_l(k))$. (b) Ensemble-average squared error curve at different values of Δf and SNR.

We empirically evaluate the mean square *a priori* estimation error given by $J'(k) = E[|r(k)/\widehat{m}(k) - V'(k)|^2]$, which yields a learning curve of the complex DA ML algorithm. The ensemble-average squared *a priori* estimation error is plotted in Fig. 1(b) for various Δf and SNR values, using a QPSK signal at $\Delta\nu$ = 2 MHz. The algorithm exhibits an exceptionally fast rate of convergence, approximately within

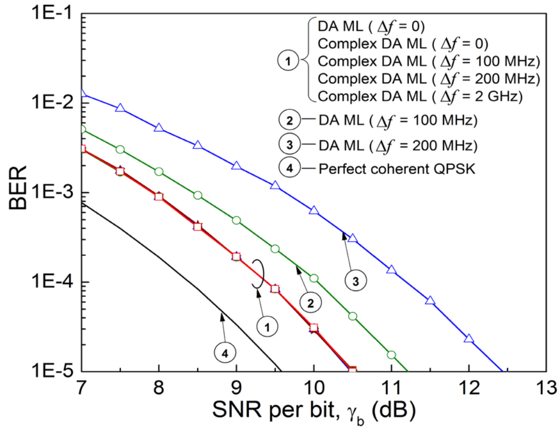

Fig. 2: BER curves of DA ML and complex DA ML receivers.

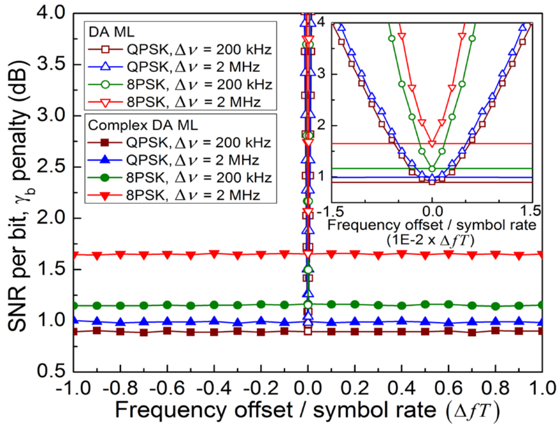

Fig. 3: Frequency offset compensation range. Inset shows enlarged $\Delta fT = [-1.5, +1.5] \times 10^{-2}$.

twice the filter length for all tested SNR values. This means that the weight $w_l(k)$ quickly achieves tracking of $exp(j2\pi l \Delta fT)$ within $2L$ iterations. Note that the mean square error does not vary with Δf, indicating that the complex DA ML algorithm is unbiased towards the frequency offset present.

Fig. 2 plots the bit-error rate (BER) curve of DA ML and complex DA ML CE for different frequency offsets, using a QPSK signal at $\Delta\nu$ = 2 MHz. DA ML CE incurs a 0.7- and 1.8-dB SNR penalty for an increase of Δf to 100 and 200 MHz, respectively. DA ML CE fails at Δf = 2 GHz. On the contrary, complex DA ML CE's BER performance when Δf = 0, 0.1, 0.2, and 2 GHz is similar to that of the DA ML receiver at zero frequency offset. Therefore, the complex DA ML CE achieves perfect frequency offset compensation at all SNR values. Converse to Kalman filter based FOE technique[8], complex DA ML CE achieves perfect compensation of Δf without statistical knowledge of AWGN, laser linewidth or frequency offset.

Fig. 3 plots the γ_b penalty at BER = 10^{-4} as Δf is swept between $\pm$20 GHz for QPSK and 8PSK signals, using $\Delta\nu$ of 200 kHz and 2 MHz. The reference is the γ_b in a perfect coherent receiver. DA ML CE is highly intolerant to frequency offsets. For a 2-dB γ_b penalty at $\Delta\nu$ = 200 kHz, a frequency offset to symbol rate ratio, ΔfT, of 7.5×10^{-3} and 2.5×10^{-3} is realized for QPSK and 8PSK signals, respectively. On the other hand, a complete frequency offset compensation range of $\Delta fT = [-1, +1]$ is achieved by our complex DA ML CE. This is attributed to the use of a RP $V'(k)$ having an unambiguous phase tracking range of $[0, 2\pi)$. Moreover, the use of a RP eliminates the need for phase unwrapping in phase noise and frequency offset compensation. There is a frequency duplicity of R but note that this duplicity does not demand differential encoding of data for correct detection at the receiver. The proposed receiver is modulation format-independent with an invariant Δf compensation range as seen from the BER performance insensitivity when compensating for Δf in QPSK and 8PSK formats. It is notable that only a 1-dB penalty is incurred at BER = 10^{-4} for a 20-Gbaud QPSK signal having a laser linewidth of 2 MHz, regardless of the frequency offset. Degradation of complex DA ML CE's BER performance is only due to laser phase noise.

Conclusion

We have presented a novel complex DA ML CE (decision-aided maximum-likelihood carrier estimation) for joint phase noise and frequency offset compensation. Complex DA ML CE has a short learning period and is adaptive to changing channel conditions. Perfect frequency offset compensation for a complete range of $\pm R$ is achieved for all signal modulation formats, where R is the symbol rate. No phase unwrapping is required.

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